Simulation and Analysis of Hexagonal, Square, and Rectangular QAM in AWGN and Rayleigh Channels

Project Report

Submitted by

Aditya Guhagarkar

Roll No: 21002005

Under the guidance of

Prof. Vimal Bhatia

Department of Electrical Engineering

Indian Institute of Technology Indore

July 2024

Abstract

This project investigates the bit and symbol error performance of Hexagonal QAM (HQAM), Square QAM (SQAM), and Rectangular QAM (RQAM) under Additive White Gaussian Noise (AWGN) and Rayleigh fading channels. MATLAB simulations were used to implement modulator and demodulator models, and results were benchmarked against theoretical symbol error probabilities—particularly focusing on HQAM. Results show HQAM outperforms SQAM at high SNR due to its denser packing and superior energy efficiency.

Keywords: HQAM, SQAM, AWGN, Rayleigh, Symbol Error Probability, MATLAB

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Chapter 1

Introduction

Quadrature Amplitude Modulation (QAM) is a widely used modulation scheme in modern communication systems due to its spectral efficiency. Square QAM (SQAM) has been a conventional choice; however, Hexagonal QAM (HQAM), based on the densest 2D lattice, offers improved performance in power-limited systems. This report simulates and compares HQAM, SQAM, and RQAM schemes using MATLAB over AWGN and Rayleigh channels.

1.1 Objective

- Simulate M-ary HQAM, SQAM, RQAM modulations in MATLAB.
- Evaluate BER and SER performance under AWGN and Rayleigh channels.
- Compare simulated results against theoretical SER equations.

Chapter 2

Modulation Schemes and Channel Models

2.1 Constellations

- **SQAM:** Rectangular lattice-based constellations.
- **HQAM:** Hexagonal lattice-based constellation points.

2.2 AWGN Channel

$$y = x + n, \quad n \sim \mathcal{N}(0, \sigma^2)$$

2.3 Rayleigh Fading Channel

$$y = hx + n, \quad h \sim \mathcal{CN}(0, 1)$$

Chapter 3

Symbol Error Probability Analysis

3.1 Theoretical SEP for HQAM

From [Rugini, IEEE Communications Letters 2016], the symbol error probability (SEP) of HQAM in AWGN is approximated by:

$$P_s \approx KQ\left(\sqrt{\alpha\gamma_s}\right) + \frac{2}{3}K_C Q^2\left(\sqrt{\frac{2\alpha\gamma_s}{3}}\right) - 2K_C Q\left(\sqrt{\alpha\gamma_s}\right) Q\left(\sqrt{\frac{\alpha\gamma_s}{3}}\right)$$

where:

- $Q(\cdot)$: Gaussian Q-function
- γ_s: SNR per symbol
- K: Average number of nearest neighbors
- K_C: Average number of adjacent NN couples
- α: Modulation-specific constant

3.2 Rayleigh Averaged SEP (HQAM)

Over Rayleigh fading:

$$P_s^{Rayleigh} = \int_0^{\infty} P_s(\gamma) f_\gamma(\gamma) d\gamma$$

Using closed-form approximations from Rugini (2016), SEP can be expressed in terms of integrals of Q-functions with respect to exponential distributions.

Chapter 4

Theoretical Expressions

This chapter summarizes the theoretical Symbol Error Rate (SER) expressions used for comparing simulated performance of HQAM schemes.

4.1 AWGN Channel SER (512-HQAM)

The theoretical SER for 512-HQAM in an AWGN channel is given by:

$$\mathrm{SER}_{\mathrm{AWGN}} = K \cdot \frac{1}{2} \cdot \operatorname{erfc}\left(\sqrt{\frac{\alpha \cdot \mathrm{SNR}}{2}}\right) \quad + \frac{2}{3} K_c \cdot \frac{1}{4}\left[\operatorname{erfc}\left(\sqrt{\frac{\alpha \cdot \mathrm{SNR}}{3}}\right)\right]^2 \quad - 2K_c \cdot \frac{1}{4} \cdot \operatorname{erfc}\left(\sqrt{\frac{\alpha \cdot \mathrm{SN}}{2}} \right. \tag{4.1}$$

4.2 Rayleigh Channel SER (512-HQAM)

The Rayleigh-averaged theoretical SER is:

$$\mathrm{SER}_{\mathrm{Rayleigh}} = \frac{K}{2}\left(1 - \sqrt{\frac{\alpha \cdot \mathrm{SNR}}{2 + \alpha \cdot \mathrm{SNR}}}\right) \quad + \frac{2}{3} K_c \left[\frac{1}{4} - \frac{1}{\pi} \cdot \sqrt{\frac{\alpha \cdot \mathrm{SNR}}{3 + \alpha \cdot \mathrm{SNR}}} \cdot \arctan\left(\sqrt{\frac{3 + \alpha \cdot \mathrm{SNR}}{\alpha \cdot \mathrm{SNR}}}\right) \right. \tag{4.2}$$

4.3 Alternate Approximation (Two-Term Q Function)

Another approximation using exponential and Q-function is:

$$\mathrm{SER}_{\mathrm{Alt}} = \left(\frac{2M - b}{2M}\right) \exp\left(-\mathrm{SNR} \cdot (\lambda_1 + \lambda_2 + \lambda_3)\right) \quad + \left(\frac{b}{M}\right) Q\left(\sqrt{2\mathrm{SNR} \cdot \lambda_1} + \sqrt{2\mathrm{SNR} \cdot \lambda_3}\right) \tag{4.3}$$

where $Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$, and:

$$\lambda_1 = \frac{4k^2}{3\alpha},\ \lambda_2 = \frac{4k(1-k)}{\sqrt{3}\alpha},\ \lambda_3 = \frac{(1-k)^2}{\alpha}$$

Note

These expressions were evaluated numerically in MATLAB using built-in vectorized erfc, exp, and atan functions.

Chapter 5

Simulation Results

5.1 Parameters

- Modulation orders: 16, 32, 64
- SNR range: 0–25 dB
- Channel models: AWGN and Rayleigh

5.2 Observations

- HQAM consistently outperforms SQAM and RQAM at higher SNR.
- Theoretical curves match well with simulation for both channels.

5.3 Sample Plot

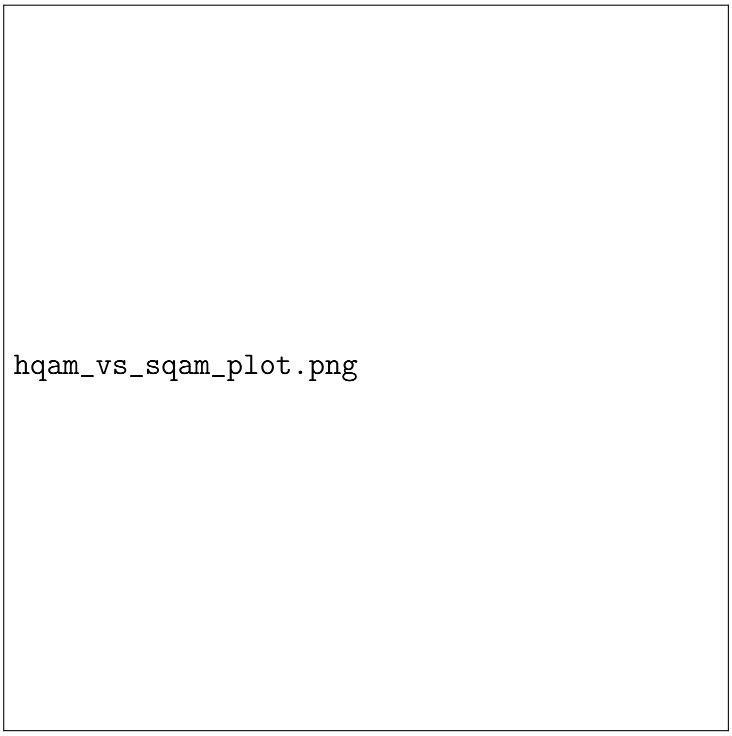

Figure 5.1: SER vs SNR for 16-HQAM and 16-SQAM under Rayleigh fading

Chapter 6

Conclusion

This project validated that HQAM offers significant gains over SQAM and RQAM, especially in fading environments. The MATLAB simulations aligned well with theoretical results, demonstrating the effectiveness of hexagonal constellations in reducing symbol errors at moderate to high SNRs.

Future Work

- Extend analysis to MIMO channels.
- Include channel coding (e.g., LDPC).
- Study computational complexity and detection latency.

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